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NATURAL CONVECTIVE HEAT TRANSFER IN VERTICAL WAVY CHANNELS

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1. INTRODUCTION

ANALYSES of fluid flow over wavy walls have applications in different areas such as transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. In view of these applications Lekoudis *et al.* [1] have made a linear analysis of compressible boundary-layer flows over a wavy wall. Shankar and Sinha [2] have studied the effects of wall waviness on the well known Rayleigh problem. Lessen and Gangwani [3] have analysed the effect of small amplitude wall waviness upon the stability of the laminar boundary layer. In all these studies the authors have taken the wavy wall to be oriented in a horizontal direction and studied the effect of the waviness on the flow field.

The present authors, Vajravelu and Sastri [4] have made a systematic analysis of free convective heat transfer in a viscous fluid confined between a long vertical wavy wall and a parallel flat wall and have established that the flow and heat transfer characteristics are significantly affected by the wall waviness. The present problem is an extension of [4] for the case when the channel walls are wavy and is taken for study for two reasons: firstly, its solution will be useful in the stability analysis (the stability results will be presented in another paper) and secondly, the heat transfer results have a definite bearing on the design of oil or gas-fired boilers. Due consideration has been given to different cases of orientation of the channel walls (see Fig. 1), because any relative differences in the orientation can lead to significant changes in the heat transfer results. The governing equations have been solved exactly analogous to that of [4]. It is interesting, but not surprising, to note that the mean part of the solution coincides with that in [4], after modifications resulting from the different choices of the origin in [4] and in the present investigation, while the perturbed part of the solution is the contribution of the waviness of the walls.

2. FORMULATION AND SOLUTION OF THE PROBLEM

Figure 1 depicts the various channels considered in this study. Let $Y = d + \varepsilon^* \cos KX$ ($=y_1$, say) and $Y = -d + \varepsilon^* \cos(KX + \omega)$ ($=y_2$, say) represent the channel walls, with ω taking values equal to 0, $\pi/2$, π and $3\pi/2$ to denote changes in the orientation of the channel walls, which are maintained at constant temperatures T_1 and T_2 respectively. Assuming the flow to be laminar, steady and two-dimensional the governing equations of the flow and heat transfer of the problem are exactly the same as those in [4]. Using the method of

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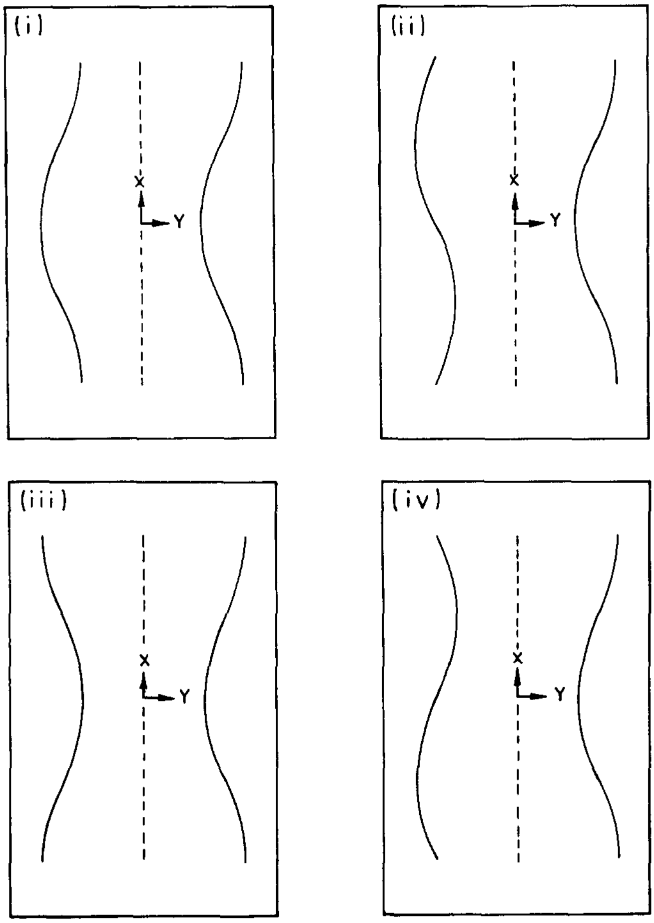

FIG. 1. Channels under consideration.

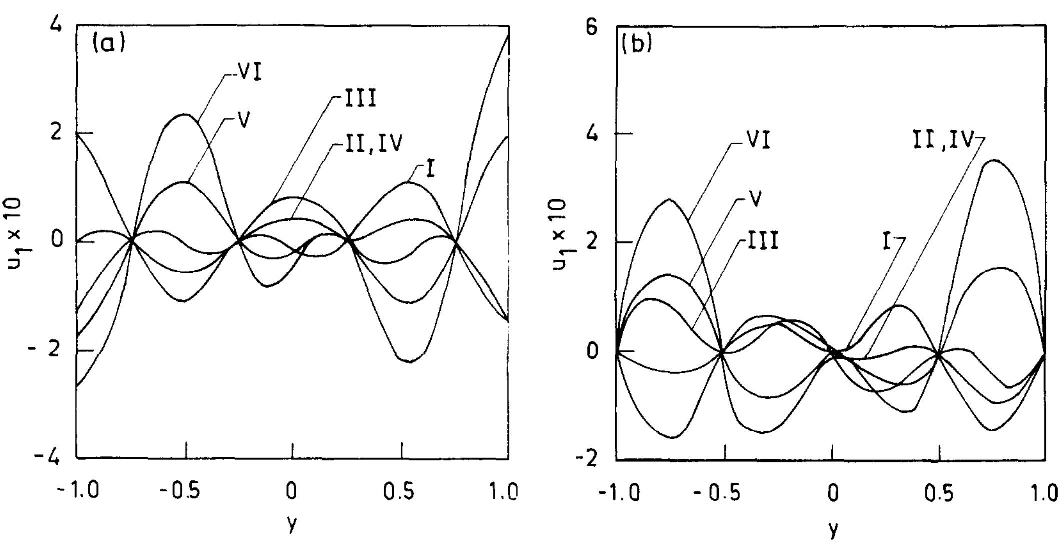

FIG. 2. Dimensionless first-order velocity profiles. $\lambda = 0.01$, $P = 0.71$ and $m = -1$. (a) $\lambda x = 0$; (b) $\lambda x = \pi/2$.

	I	II	III	IV	V	VI
G	5	5	5	5	5	10
α	-5	-5	-5	-5	5	5
ω	0	$\pi/2$	π	$3\pi/2$	0	0

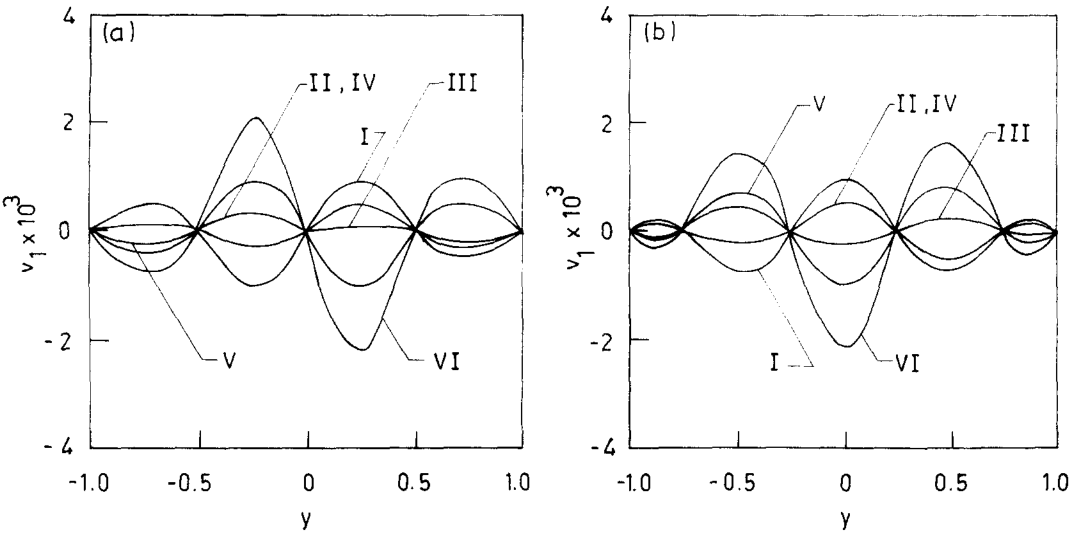

FIG. 3. Dimensionless first-order velocity profiles. Legends and curves as in Fig. 2.

perturbations we have taken the flow and the temperature-fields in the non-dimensional variables as

$$u(x,y) = u_0(y) + u_1(x,y), \quad v(x,y) = v_1(x,y),$$

$$\bar{P}(x,y) = \bar{P}_0(x) + \bar{P}_1(x,y), \quad \theta(x,y) = \theta_0(y) + \theta_1(x,y)$$

and obtained the solutions for the mean parts (u_0, θ_0) and the perturbed parts (u_1, v_1, θ_1, $\bar{P}_1$) subject to the relevant boundary conditions (for details, the reader may refer to [4]). For the sake of brevity, the solutions are not presented here.

As in [4], here also, the skin friction coefficients $\tau_{1,2}$ and the heat transfer coefficients $Nu_{1,2}$ have been obtained along with the pressure drop $\hat{P}$. These flow and heat-transfer characteristics have been found to depend on the dimensionless parameters G, the free convection parameter; λ, the frequency parameter; m, the wall temperature ratio; P, the Prandtl number; ε, the amplitude of the wavy wall and α, the heat source/sink parameter and evaluated numerically for several sets of values of these parameters, in addition to x and y. These results are embodied in Figs. 2–5. In Section 3 we have recorded the qualitative behaviours of the flow and heat transfer characteristics which show the effect of the waviness of the channel walls.

3. DISCUSSION OF THE RESULTS

In [4] a detailed account has been given as to the behaviour of the mean part of the solution and those results hold more or less qualitatively here as well. In what follows the discussion has been restricted to the perturbed parts of the solution in the case of air ($P = 0.71$) when $m = -1$ only, as the results are qualitatively applicable in the case of water ($P = 7$).

Figures 2, 3 and 4 describe the behaviour of the perturbed quantities u_1, v_1 and θ_1, when $\lambda x = 0$ and $\pi/2$, respectively. From Fig. 2 it is clear that u_1 is affected significantly by the parameter ω indicating that u_1 is quite different in the several channels under consideration. It is also evident that the velocity u_1 increases with the free convection parameter G or the wall temperature ratio m or the heat source parameter α, this result holding even with the frequency parameter λ. A close look into Fig. 2 reveals further that the perturbed velocity is oscillatory in nature and this nature is more prominent when the parameters ε, G and α take higher values.

Figure 3 shows the behaviour of v_1, the fluid velocity perpendicular to the channel length. A comparative study of Figs. 2 and 3 reveals that the qualitative behaviour of v_1 is similar to that of u_1 despite the fact that the values of v_1 are

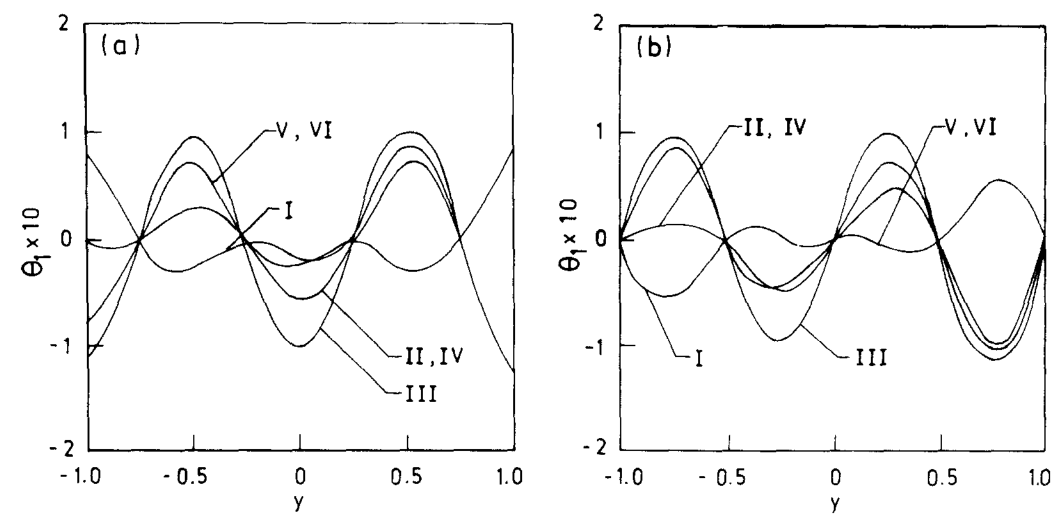

FIG. 4. Dimensionless first-order temperature profiles. Legends and curves I-VI as in Fig. 2.

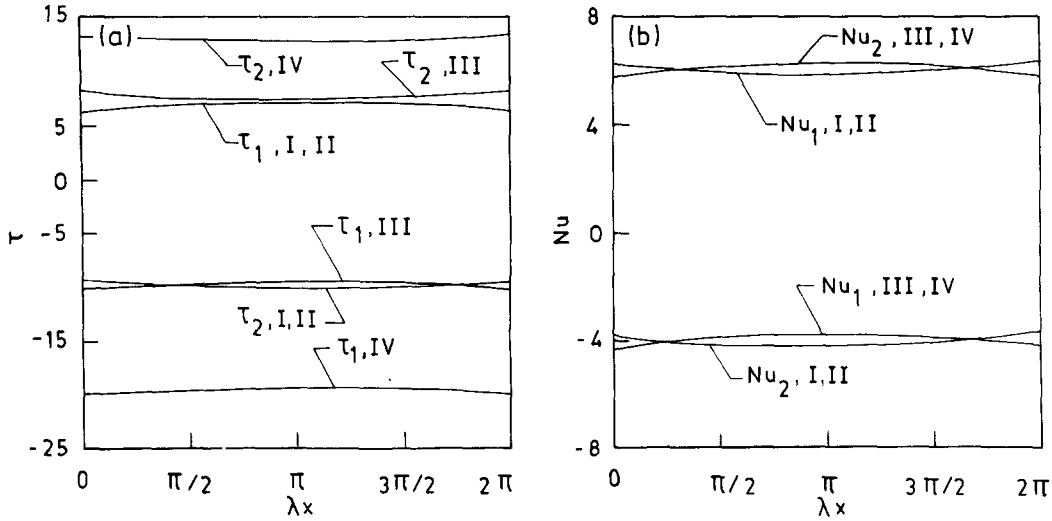

FIG. 5. Flow and heat transfer characteristics at the walls. $\lambda = 0.01$, $P = 0.71$ and $m = -1$. (a) Skin friction; (b) Nusselt number.

	I	II	III	IV
G	5	5	5	10
α	-5	-5	5	5
ω	0	$\pi/2$	0	0

much smaller than those of u_1. From Fig. 4 it is noticed that the heat source parameter α has the strongest effect on the temperature θ_1 and that the effect of wall waviness on θ_1 is very much similar to that on u_1.

It is worth noting that the effects of the various parameters on the total velocities (u, v) and the total temperature (θ) are qualitatively similar to those on their perturbed counterparts.

Figure 5(a) shows the behaviour of the skin friction at the channel walls. From this figure it is clear that, the skin friction at the wall $y = y_2$ increases with G and α, this increase with G being the greater; nevertheless, this behaviour is reversed at the other wall. Figure 5(b) describes the behaviour of the wall heat transfer coefficients. From Fig. 5(b) it is noticed that, the heat transfer coefficient at the wall $y = y_2$ increases with α, G and ω, while the reverse is true at the other wall. Also, when the heat source parameter α takes increasing positive values, the heat transfer coefficient is found to be positive at the wall $y = y_2$ and negative at the other wall, which result indicates physically that the heat flows into the walls only. However, this behaviour is duly reversed in the case of heat sinks ($\alpha < 0$). Based on the numerical calculations, we finally conclude that, in the presence of heat sinks, the pressure at $y = y_2$ exceeds that on the other wall in the first half ($0 \leq \lambda x \leq \pi/2$), while when $\alpha > 0$, this behaviour gets reversed.

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